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A CRITERION FOR QUASIBRITTLE CRACK GROWTH

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INTRODUCTION

In this paper we derive an expression to predict the onset of growth of a crack in a quasibrittle material. We employ an energy criterion, which was first proposed by Griffith (1921), and is essentially equivalent to the first two laws of thermodynamics. The criterion is applied to the elastic-plastic crack of Olesiak and Wnuk (1968), which is a three-dimensional generalization of the Dugdale crack (1960).

Our expression for the onset of growth differs in several respects from a criterion based on a crack in a brittle material (i.e., using linear elastic fracture mechanics). Furthermore, the new features are in qualitative agreement with the experimental data. Among these new features are

- 1) the dependence of the apparent surface energy, or critical strain energy release rate on crack size (the shape of the R-curve);
- 2) the effect of in-plane stresses on crack growth (i.e., the stresses that do not lead to any traction on the crack faces); and
- 3) a brittle-ductile transition.

The energy criterion, as a necessary condition for crack growth, rests on a firm theoretical basis. However, its role as a sufficient condition is less secure. Several alternate criteria have been proposed as necessary and sufficient conditions for the onset of crack growth. We will contrast several of these, including critical strain (Olesiak and Wnuk, 1968; Goodier and Field, 1963) and crack opening displacement (Burdekin and Stone, 1966) with the energy criterion. We will show that these alternate approaches are inconsistent with the energy criterion, in the sense of being necessary conditions, and so should be rejected as criteria for growth--at least for the elastic-plastic crack.

GROWTH CRITERIA

In this section we briefly review aspects of some of the criteria proposed for crack growth. A more complete treatment can be found in (Knott, 1973; Nichols, 1979). As a statement of the problem, consider a crack, ideally penny-shaped, and embedded in a body whose dimensions are much larger than the crack. A uniform stress is applied to the surface of the body. We require a criterion to predict the onset of crack growth. The criterion, when applied to a specific model of the crack, will yield a critical crack size (for growth) as a function of the applied stress and the material properties.

In 1921, Griffith applied the idea of an energy balance to the problem of predicting crack growth. In general terms, Griffith's energy criterion states that a crack will grow if the potential energy released by that growth exceeds the energy dissipated during growth.

Griffith identified the potential energy with the stored elastic strain energy. The energy dissipated by breaking bonds at the crack tip was represented macroscopically by an energy to create new surface. For penny-shaped cracks, the strain energy scales like crack length cubed (l^3) whereas the energy to create new surface scales like crack length squared (l^2). Application of the energy criterion then leads to the inequality

$$\sigma_{zz}^2 > \frac{\sqrt{TE}}{2(1-\nu^2)l} \quad (1)$$

We emphasize that Eq. 1 is only one realization of the energy criterion, based on a particular model of the energies involved in growth. The energy criterion is much more general than Eq. 1, and is, in fact, based on the first two laws of thermodynamics (Eftis and Liebowitz, 1975; Nichols, 1979). However, the energy criterion is a global criterion and (like thermodynamics) makes no statement about sufficient conditions for crack growth. To construct a sufficient condition, one must decide on a mechanism of crack growth. Several such criteria have been devised, each focusing on the details of the crack tip.

However, the crack tip is a region of high stress concentration. It is precisely the region where the crack models based on elastic fracture mechanics break down, and where inelastic processes occur. Irwin (1957) and Orowan (1955) pointed out that the crack tip must be a region of plastic flow. They accounted for this extra dissipation by postulating an effective surface energy coefficient to include both breaking atomic bonds and plastic work. Inherent in this approach is the assumption that the new surface energy coefficient is a material constant. From this point, the theory closely resembles Griffith's analysis.

Another approach discards the brittle crack for the Dugdale (1960) crack. Here, plastic extensions of the crack remove the elastic singularities found at the tip of the brittle crack. Criteria for growth are then postulated (with no theoretical basis) as either a critical displacement or a critical strain at the crack tip.

Goodier and Field (1963) calculate the plastic work at the tip of a two-dimensional crack; Olesiak and Wnuk (1968) do the calculation for a three-dimensional crack. In each case, the effective surface energy coefficient due to plastic work is not constant, but scales like crack length (l). Also, the plastic work scales in a complicated way with the applied stresses.

In this paper, we apply the general energy criterion to the three-dimensional crack of Olesiak and Wnuk. One of the interesting results is to demonstrate that the COD and the critical strain criteria, postulated as necessary and sufficient, actually violate the necessary condition based on thermodynamic considerations.

THE ENERGY CRITERION

The model developed by Olesiak and Wnuk (2) is shown in Fig. 1. The crack surface has boundary conditions

$$\begin{aligned}\sigma_{zz} &= -p_0 & 0 < r < l \\ \sigma_{zz} &= -(p_0 - Y) & l < r < a\end{aligned}\tag{2}$$

where a is the total length of the crack and the region $l < r < a$ is the plastic extension. The length l is chosen to remove the stress singularity at the crack tip. This corresponds to case 2, both in (Olesiak and Wnuk, 1968) and in (Wnuk, 1968); that is, a tensile stress p_0 applied at infinity.

Here also Y is the plastic yield and so $|\sigma_{zz}| < Y$ everywhere. Note that the plasticity condition only enters through the boundary condition, and that the elastic-plastic problem has been reduced to one that can be treated in the framework of elasticity.

To apply the energy criterion, we must next identify the various energy terms. The potential energy is still the elastic strain (W_e). The dissipated energy will be the Griffith surface energy (W_g) plus the plastic work (W_p) done at the crack tip. Wnuk (1968) calculates, for the quasibrittle crack,

$$W_e = \frac{8(1-\nu^2)l^3Y^2}{3E} \cdot 2 \cdot (1 - \sqrt{1-\lambda^2})\tag{3}$$

$$W_p = \frac{8(1-\nu^2)l^3Y^2}{3E} \cdot 2 \cdot \frac{(1 - \sqrt{1-\lambda^2})^2}{\sqrt{1-\lambda^2}}\tag{4}$$

where $\lambda = p_0/Y < 1$. For the surface energy, we take the Griffith form

$$W_s = 2\pi\Gamma\ell^2 \quad (5)$$

where Γ is the free-surface energy per unit area. Assuming for the moment that the external boundaries of the body maintain a fixed displacement during crack growth (so there is no change in the potential energy of the loading mechanism), the energy criterion is

$$\frac{\partial}{\partial \ell} (W_e - W_p - W_s) > 0 \quad (6)$$

Substituting from Eqs. 3 and 4, we obtain

$$\left[(1 - \sqrt{1-\lambda^2}) - \frac{(1-\sqrt{1-\lambda^2})^2}{\sqrt{1-\lambda^2}} \right] > \frac{\pi\Gamma E}{8(1-\nu^2)Y^2\ell} \quad (7)$$

ANALYSIS

First, consider the limit $Y \rightarrow \infty$. In this limit, the plastic zones vanish and we regain the elastic crack. Equation 6 then should limit to Eq. 1 and the growth criterion should become independent of Y ; this is easily verified.

Second, we note that the left-hand side of Eq. 6 has a maximum. This occurs at

$$\sqrt{1-\lambda^2}_{\max} = \frac{1}{\sqrt{2}} \quad \text{or} \quad \lambda_{\max} = \frac{1}{\sqrt{2}} \quad (8)$$

$$\text{Equation 8 says that when } \sigma_{zz} = 0.7071 Y \quad (9)$$

then the smallest crack that may grow under any circumstances will commence to grow. Substituting Eq. 9 into Eq. 6, we derive the minimum length crack liable to growth

$$\ell_{\min} = \frac{2.29 \Gamma E}{(1-\nu^2) Y^2} \quad (10)$$

Note that there is no minimum crack liable to growth in Griffith theory.

Next, we note that there is a maximum value of λ beyond which plastic work accumulates faster than strain energy is released. This occurs when

$$\lambda_{\max} = \sqrt{3/4} \quad \text{or} \quad \sigma_{zz(13)} = 0.866Y \quad (11)$$

The meaning of this limit is that beyond the critical value of tensile stress given by Eq. 11, no crack can grow.

The growth criterion in general, as represented by Eq. 7, is shown graphically in Fig. 2. In this figure, the dimensionless normal stress is plotted against the normalized critical crack length ($\alpha = l/l_{\min}$) (see Eq. 10). The range of crack length is divided into two regions--Region I where failure can occur by crack propagation, and Region II where failure occurs by "plastic" collapse. (We do not infer from our calculations that the cracks of Region II are really stable, but rather that our model, based on the growth of long, thin cracks by extension at the tip, has broken down.)

The dotted line in Fig. 2 shows the Griffith criterion for brittle cracks (i.e., for $Y \rightarrow \infty$). We can define an effective surface energy from the quantity σ_{zz}/\sqrt{l} . For the Griffith crack, this quantity is a constant independent of crack length, from which the surface energy Γ may be determined.

In nondimensional terms, we plot λ/α versus α . This may be interpreted as the ratio of an effective surface energy Γ_E , to the Griffith surface energy as a function of crack size. The graph, shown as Fig. 3, demonstrates that the effective surface energy approaches the Griffith value, being higher for smaller cracks.

This last point merits some discussion. Because surface energy scales like [crack length squared], and the plastic work like [crack length cubed], one might expect the plastic work to be most significant for big cracks [see, for example, the comments on p. 112 of (Eftis and Liebowitz, 1975)]. Such a conclusion ignores the complicated dependence of the plastic work on the stress. Larger cracks correspond to lower values of critical stress. In fact, the manner in which the plastic work decreases with decreasing stress dominates the increase of plastic work with crack length. The proof of this statement is, of course, Fig. 3.

OTHER CRITERIA

The displacement at the tip of the three-dimensional elastic-plastic crack (Olesiak and Wnuk, 1968) is

$$w(l, l) = \frac{4(1-\nu^2)lY}{\pi E} (1 - \sqrt{1-\lambda^2}) \quad (12)$$

The COD criterion for crack growth would be $w(l, l) < d$ (13)

where d is a material property. This can be written

$$(1 - \sqrt{1-\lambda^2}) < \frac{d\pi E}{4(1-\nu^2)\ell Y} \quad (14)$$

However, Eq. 6 can be written

$$(1 - \sqrt{1-\lambda^2}) < \frac{2\pi\Gamma E}{16(1-\nu^2)Y\ell} + \frac{1 - \sqrt{1-\lambda^2}}{\sqrt{1-\lambda^2}} \quad (15)$$

Since Eq. 15 represents a necessary condition and Eq. 13 purports to represent a necessary and sufficient condition, we must have

$$\frac{d\pi E}{4(1-\nu^2)\ell Y} < \frac{\pi\Gamma E}{8(1-\nu^2)Y\ell} + \frac{1 - \sqrt{1-\lambda^2}}{\sqrt{1-\lambda^2}} \quad (16)$$

Now d is supposed to be material constant, independent of crack size (ℓ) and loading (λ). However, the right side of Eq. 16 is unbounded as $\lambda \rightarrow 1$. Thus, it is not possible to choose any finite value of d such that Eq. 16 will hold for all physical possible loadings. That is, the COD criterion is not consistent with the energy criterion.

One should not view this as a contradiction between two competing theories. Rather, one must conclude that the existence of a critical opening displacement as a material property, and its use as a growth criterion, violate the laws of thermodynamics, and must be rejected for the Dugdale crack.

The critical strain criterion would be stated (refer to Eq. 13) as

$$\frac{w(\ell, \ell)}{\ell} < \epsilon_L \quad (17)$$

or

$$1 - \sqrt{1-\lambda^2} < \frac{\pi E \epsilon_L}{4(1-\nu^2) Y} \quad (18)$$

A similar argument shows that the existence of a critical strain, ϵ_L , as a material property and its use as a growth criterion would also violate the first two laws of thermodynamics.

EFFECT OF THE IN-PLANE STRESSES

So far we have considered the boundary condition

$$\sigma_{zz} = -p_0 \quad (19)$$

far from the crack. Suppose now that we also apply uniform stresses σ_{xx} and σ_{yy} at infinity.

For the elastic crack, we may simply superpose these stresses everywhere. These components do not lead to any traction on the crack faces, so the uniform field satisfies boundary conditions on the crack and at infinity. Furthermore, the strain energies associated with σ_{xx} and σ_{yy} do not depend on crack length. The net result is that the criterion for growth of the elastic crack (Eq. 1) is independent of the in-plane stress components σ_{xx} and σ_{yy} .

The situation is different for the elastic-plastic crack. Although the in-plane stresses still lead to no traction, they do affect the yield condition. In (2), the yield condition is simply,

$$|\sigma_{33}| < Y \quad (20)$$

The in-plane stresses alter this. For example, if we assume a von Mises yield condition

$$(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{xx})^2 < 2Y^2 \quad (21)$$

where Y is a material property, then the effective yield Y_{eff} that must be used in the boundary conditions is

$$|\sigma_{zz}| < Y_{eff} \quad (22)$$

where

$$Y_{eff} = \sqrt{Y^2 - \frac{3}{4}(\sigma_{xx} - \sigma_{yy})^2} + \frac{1}{4}(\sigma_{xx} + \sigma_{yy}) \quad (23)$$

The solution is the same as given in (Olesiak and Wnuk, 1968) except that now we must use Y_{eff} instead of Y , and

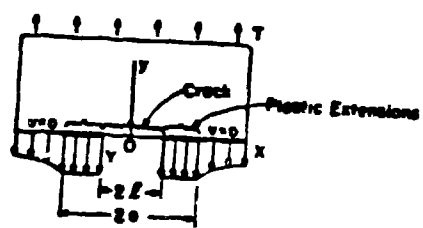
$$\lambda = \frac{\sigma_{zz}}{Y_{eff}} \quad (24)$$

The net result is that the in-plane stresses do affect the strain energy and plastic work, and now enter into the growth criterion.

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(a)

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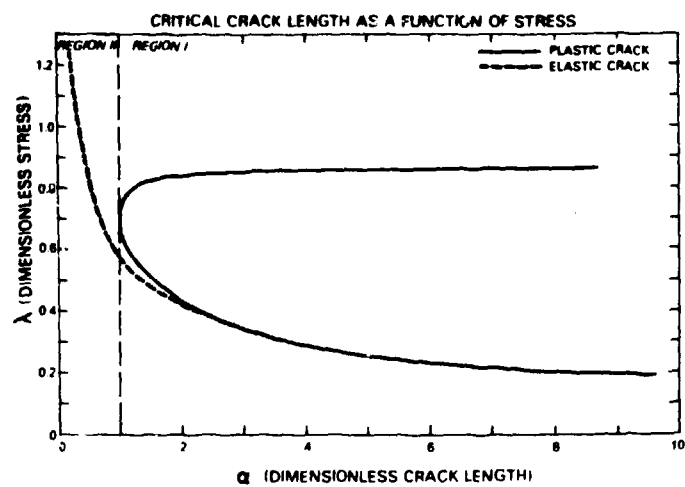


Fig 2

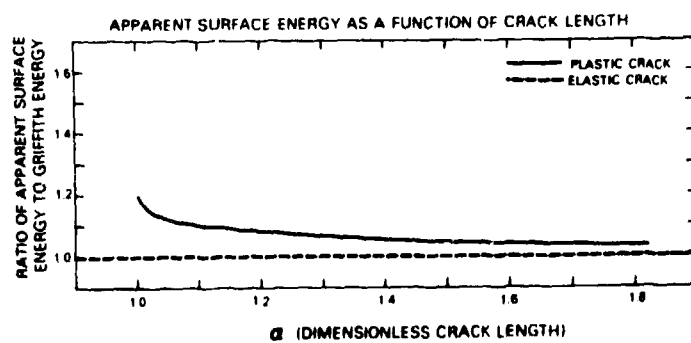


Fig 3